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Telecommunication Networks Group

Comparison of Different Fairness Approaches in OFDM-FDMA Systems

James Gross, Holger Karl

{gross,karl}@tkn.tu-berlin.de

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Abstract

Dynamically assigning subcarriers of wireless OFDM systems to terminals is a promising method for fine-grained resource management; this method can also exploit the wireless channel's variations to improve, for example, transmission capacity. However, a pure capacity maximization approach will lead to a highly unbalanced rate allocation per terminal where terminals close to the access point receive most or all of the capacity of the cell. Therefore, some form of *fairness* has to be introduced.

In this paper we study two fairness models: A subcarrier fair one, balancing the resources, and a data-rate fair model, balancing the utility of the resources. Within these constraints, capacity optimization is still necessary – we investigate the capacity-optimal solutions of linear programs and compare the models in terms of average capacity achieved per cell as well as fairness achieved in terms of equal average throughput among all terminals in the cell. We find that for a given terminal number in a cell there exists a characteristic cell radius up to which both models achieve an almost equal performance in terms of capacity and throughput fairness. Beyond this radius the subcarrier fair approach becomes more and more "unfair" in terms of equal throughput per terminal compared to the data-rate fair model.

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Chapter 1

Introduction

Orthogonal **F**requency **D**ivision **M**ultiplexing (*OFDM*) is an attractive technology for wireless communication as it allows a fine-grained adaptation of the available resources: OFDM divides a given amount of radio spectrum into "subcarriers" that can be individually assigned to different terminals for the downlink communication from an access point to the terminals in a wireless cell. This flexibility can be used, e.g., to choose subcarrier-terminal assignments that maximize the capacity of a wireless cell by assigning a given subcarrier only to terminals for which it is in a good state, meaning that many bits can be transmitted via this subcarrier.

Such a simple maximization, however, would likely starve terminals that are far away from the access point: There would typically be, for each subcarrier, a terminal closer to the access point that could realize a higher data-rate over this subcarrier than a far-away terminal. Therefore, an imbalance is created in terms of assigned resources per terminal – which, in such a system, are the subcarriers – as well as utility of these resources per terminal – the overall data-rate resulting from the assigned subcarriers per terminal – between terminals close to the access point and ones which are far away.

In order to prevent these imbalances, two different solutions of introducing "fairness" are possible: One could try to achieve fairness by dividing the resources equally among the terminals [1, 2], thus resulting in *subcarrier fairness*. Or one could try to divide the utility – the number of bits transmitted per downlink phase – equally over the terminals [3, 4], resulting in *data-rate fairness*. Obviously, the first approach is simpler to realize from the system management point of view, while the second one is more desirable from the terminals' point of view.

In this paper we compare these two approaches, looking at the total capacity obtained per cell as well as at the extent with which the utility-oriented fairness can be approximately achieved by the fair division of resources. The system model we use for this investigation is described in Section 2, the two fairness concepts are formalized in Section 3 as linear programs. We consider the optimal solutions to these linear programs in order to compare the two fairness models in Section 4; we varied parameters like cell size and terminal number to gain a better understanding of the problem space. Finally, we conclude the paper in Section 5.

Chapter 2

System Model

Let us assume a wireless cell of radius R with J wireless terminals in it. Any data transmission within this cell is managed by an access point. We only consider the downlink transmission of data from the access point to the terminals. Time is slotted into units of length T, called a **M**edium **A**ccess **C**ontrol (*MAC*) frame, where T is split into two equally long phases, one for downlink and one for uplink transmission.

For data transmission a spectrum of bandwidth B is provided with a center frequency of f_{c}. OFDM is used to split the bandwidth into S subcarriers where data might be transmitted on each one in parallel. Any two adjacent subcarriers are spaced apart by $\Delta f = \frac{1}{T_s}$, where T_s denotes the time span of one symbol.

Each subcarrier has a constantly varying attenuation with respect to each wireless terminal. These attenuation states are determined by path loss, shadowing and fading due to multi-path propagation and mobility. More precisely, $a_{j,s}(t)$ models the attenuation of subcarrier s between the access point and the wireless terminal j at time t. These attenuation states are summarized in a matrix $A(t) = (a_{j,s}(t))$. The subcarrier attenuation states are correlated both in time and frequency, characterized by the power delay profile and power spectrum of the fading. The subcarrier attenuation states are assumed to remain constant during a MAC frame of length T

Per subcarrier the same transmit power is applied per downlink phase. This transmit power is given by P_{tx}. The noise power per subcarrier is given by n_0. The actual **S**ignal–to–**N**oise **R**atio (*SNR*) per subcarrier $x_{j,s}(t)$ is hence given by Equation 2.1.

$$x_{j,s}(t) = P_{\mathrm{tx}} \cdot \frac{a_{j,s}^2(t)}{n_0} \tag{2.1}$$

We assume the access point to have knowledge of these SNRs prior to each downlink phase. This knowledge enables dynamic OFDM-FDMA to combat the variations of subcarrier attenuation states: For each downlink phase the set of S subcarriers is divided into disjunctive subsets of subcarriers where terminals receive their data on these subcarrier subsets exclusively. The generation of the subsets is based on the provided subcarrier state information and is executed by an assignment algorithm. A separate control channel is used to signal the assignments to the terminals prior to each downlink phase.[1] The subcarrier assignments are valid during an entire downlink phase.

In addition to the adaptive subcarrier allocation also adaptive modulation is applied in the system. Depending on the subcarrier state a different modulation type is chosen such that the resulting amount of data which can be transfered per downlink phase varies from subcarrier to subcarrier. The adaptive modulation scheme is given by the function $F(x_{j,s}(t))$, returning the number of bits which might be transmitted on subcarrier s to terminal j during the downlink phase at time t. $F(\cdot)$ is assumed to return exactly one modulation type for any SNR input variable.

At the access point data arrives constantly from a backbone destined for each wireless terminal, where the data-rate of each stream is equal. Each terminal has its own queue within the access point. The queues are assumed to be never empty, the access point holds always some data that has to be transmitted to each terminal in the cell.

The overall objective in the cell is to find assignment schemes which maximize the capacity in the cell. We can formulate an optimization problem for this, given in Equation 2.3. $c_{j,s}(t)$ is a binary variable and denotes whether or not subcarrier s is assigned to terminal j at time t. $C(t)$ is the corresponding matrix denoting all binary assignments $c_{j,s}(t)$.

$$\begin{aligned} \max_{C(t)} \quad & \textstyle\sum_{\forall j,s} c_{j,s}(t) \cdot F(x_{j,s}(t)) \\ \text{s.t.} \quad & \forall s : \textstyle\sum_{\forall j} c_{j,s}(t) \leq 1 \end{aligned} \tag{2.2}$$

The constraint of this optimization problem ensures that each subcarrier is assigned either once or not at all, since for one downlink phase only one terminal can receive data on a subcarrier.

[1] We ignore the impact of this signaling overhead here and investigate it in a separate paper.

Chapter 3

Fairness Models

The optimization problem in Equation 2.3 is simple to solve: always assign a subcarrier to the terminal which has the best state for it. By this the capacity is maximized.

However, terminals are distributed over the whole cell. Some terminals are close to the access point, some are quite far away. Therefore, some will experience quite good subcarrier states overall (since they are close) while other, far away terminals will experience quite poor subcarrier states overall. As a consequence, using the above mentioned solution for capacity optimization, the terminals far away from the access point will receive only very little capacity while the terminals close to the access point receive most of the capacity. Despite this discrepancy, data for all terminals is constantly received at the access point and therefore has to be transmitted to these terminals. An algorithm solving only the optimization problem of Equation 2.3 will not meet this requirement of all terminals. What has to be introduced instead is some form of *fairness* between the terminals in the cell.

Due to the traffic model where each terminal's stream has the same rate with which data arrives at the access point obviously the desired fairness from the point of view of the terminals in the cell is *data-rate fairness*. Thus, it is of interest to transmit the same amount of data to each terminal in the cell per downlink phase.

As an alternative, we also study a model providing fairness in terms of subcarriers assigned to each terminal, called *subcarrier fairness*. In this case all terminals are assigned an equal amount of subcarriers for each downlink phase regardless of the amount of bits that might be transmitted during each downlink phase to each terminal.

For both fairness models the assignment which yields the highest capacity is of interest. Both fairness approaches can be formalized in terms of an optimization problem. In the case of the subcarrier fairness model the optimization problem is given by Equation 3.1.

$$\begin{aligned}\max_{C(t)} \quad & \sum_{\forall j,s} c_{j,s}(t) \cdot F\left(x_{j,s}(t)\right) \\ \text{s.t.} \quad & \forall s : \sum_{\forall j} c_{j,s}(t) \leq 1 \\ & \forall j : \sum_{\forall s} c_{j,s}(t) = \text{ const.}\end{aligned} \tag{3.1}$$

In the case of the data-rate fairness model the optimization problem is given by Equation 3.2. The intuition behind Equation 3.2 is this: First of all a maximum lower data limit ε is of interest which can be transmitted to each terminal. This data amount ε is limited by some or multiple terminals which are called "critical" terminals. If this limit is found, multiple assignment matrices might still be feasible. Then, the one matrix is of interest which maximizes the transmitted amount of bits, hence the capacity. However, the optimization's primary target is the maximum lower limit. Therefore, the achieved capacity is scaled by ω which turns the achieved capacity into a number lower than one – the integer solution is then primarily determined by ε. Equation 3.2 is a modified formulation from the one given in [3].

$$\begin{aligned}\max_{C(t)} \quad & \varepsilon + \left(\sum_{\forall j,s} c_{j,s}(t) \cdot F\left(x_{j,s}(t)\right) \right) \cdot \omega \\ \text{s.t.} \quad & \forall s : \sum_{\forall j} c_{j,s}(t) \leq 1 \\ & \forall j : \sum_{\forall s} c_{j,s}(t) \cdot F\left(x_{j,s}(t)\right) \geq \varepsilon\end{aligned} \tag{3.2}$$

Compared to the optimization problem given in Equation 2.3, requiring data-rate fairness increases the complexity of the problem and will also, in most cases, reduce the overall achieved capacity. From the application layer point of view, what is most important is the optimal solution to problem 3.2, since it tries to achieve for each terminal a conveyable number of bits per downlink phase which is equal or higher than the maximal lower limit. However, since the capacity-optimal solution of Equation 3.2 is of interest, not all terminals will necessarily receive always the exact same number of bits per downlink phase. The reason for this is related to the "critical" terminal: this terminal can not receive a higher number of bits during the observed downlink phase without turning a different terminal into the "critical" one. With other words, even if other terminals receive a significantly higher number of bits than ε this additional amount of bits can not be given to the "critical" one without turning a different terminal into the "critical" one with a lower ε than before. As a consequence, there are multiple matrices $C(t)$ solving the second constraint of Equation 3.2, which might all yield ε bits conveyable to the "critical" terminal in the cell. However, between these different matrices there exist still capacity differences for the remaining terminals. Therefore, we are interested in the one yielding the maximum capacity. As stated this might lead to a certain imbalance again, however this imbalance can only be avoided by artificially reducing the transmittable number of bits of the “non-critical” terminals, without any effect for the “critical” terminals (otherwise for this problem instance the limit ε would be higher right away).

Intuitively, the achieved capacity in the case of subcarrier fairness is between the optimal solution of problem 2.3 and the data-rate optimal solution of problem 3.2[1]. The same number of subcarriers can always be assigned to each terminal, so the fairness achieved by this model is always perfect. This is obvious for cases where J is a multiple of S, but even in all other cases subcarrier fair assignments can be found by either not assigning all subcarriers in the system or by distributing this subcarrier remainder over different terminals from time to time, such that over a larger time span the number of assigned subcarriers is equal per terminal.

For the subcarrier fairness the throughput per terminal might vary quite strongly depending on the cell size, the path loss model and the transmitted power at the access point. Therefore, the desired equal throughput for different terminals can not be guaranteed in general using the subcarrier fairness model. On the other hand, if by path loss for example the average SNR difference between closer terminals and farther terminals is not too significant, using subcarrier fairness might achieve also quite good results in terms of throughput fairness.

[1]Also, both problems differ in terms of their computational complexity and the run-time behavior of their solving algorithms; however, this is not considered in this work

Chapter 4

Performance Evaluation

In order to study the behavior of the two different fairness models we conducted the following investigation. While keeping the transmission power and path loss model fixed, we increased the radius of the cell R and simulated the performance of the two approaches by generating many consecutive SNR matrices and solving the linear programs of both fairness models for all matrices. In addition, we chose a special positioning of the terminals in the cell, corresponding to a uniform distribution of the terminals over the *area* of the cell. The terminals were positioned along the cell's radius, such that for each adjacent pair of terminals Equation 4.1 holds. d_j denotes the distance between the access point and terminal j and therefore denotes the position of terminal j along the cells radius. As only the distance from the access point is relevant to compute SNR values, we can safely ignore the direction of a terminal's position.

$$\begin{aligned} \forall j \in \{1, \ldots, J\} : \pi \cdot \left(d_j^2 - d_{j-1}^2\right) \\ = \pi \cdot \left(d_{j+1}^2 - d_j^2\right) = \pi \cdot \frac{R^2}{J} \end{aligned} \quad (4.1)$$

This method yields an "equi-areal" positioning: the assumption is that terminals are distributed with a uniform density instead of using an equi-distant positioning which would represent a uniform distribution of the terminals' distances along the radius, giving a larger than expected share of close-by terminals. By keeping the terminals at these positions the average SNR spread between the closest and the farest terminal with respect to the access point constantly increased when increasing R (since the path loss discrepancy between all terminals increased) and therefore achieving fairness in terms of throughput became more and more difficult for any approach.

For each radius R, we simulated different numbers of wireless terminals in the cell, namely $J = 4, 6, 8, 12, 16, 24$ were simulated and the average throughput achieved per wireless terminal was recorded. The summation of these recorded average throughputs per terminal represented the capacity achieved by a certain fairness model for a given value pair (R, J). In addition, we calculated a measure to judge the throughput fairness of each method in each specific cell setting. For this, the coefficient of variation of the average throughputs per terminal was used: We calculated the standard deviation of the different average throughputs per terminal for a specific setting of J and R and divided this by the overall average throughput per terminal in this case. This was conducted for the data-rate fairness model as well as for the subcarrier fairness model for all settings of J and R.

4.1 Scenario

For the simulation environment we chose the following settings. The available bandwidth of the cell equaled $B = 16.25$ MHz, the number of available subcarriers was $S = 48$, equivalent to systems such as IEEE 802.11a [5]. Accordingly, the symbol length of one OFDM symbol was chosen to be $4\ \mu\text{s}$. As center frequency we assumed $f_c = 5.2$ GHz, thus picking the lower U-NII frequency band ($5.15 – 5.25$ GHz) of IEEE 802.11a. Such a system employs in this frequency band a total transmission power of 40 mW, which corresponds to a transmission power of $P_{\text{tx}} = -7$ dBm per subcarrier.

The adaptive modulation scheme utilized five different modulation types: BPSK, QPSK, 16-QAM, 64-QAM and 256-QAM. A modulation type was chosen if it provided the highest bit rate while keeping the symbol error probability below 10^{-2}, given the actual subcarrier's SNR $x_{j,s}(t)$. T was set to 2 ms, therefore the algorithms generated every 2 ms new subcarrier assignments from the current attenuation matrix.

The cell's radius R was varied beginning with 10 m up to a maximum of 200 m. The step interval was equal to 10 m.

The subcarrier attenuation states changed due to path loss, shadowing and fading. Path loss was determined by the formula $\frac{P_0}{P_{\text{tx}}} = K \cdot \frac{1}{d^\alpha}$, where $\frac{P_0}{P_{\text{tx}}}$ denotes the ratio between received and transmitted power, d denotes the distance between transmitter and receiver, K denotes the reference loss for the distance unit d is measured in and α is the path loss exponent. We parameterized the reference loss with $10\ \log(K) = 46.7$ dB and the path loss exponent with $\alpha = 2.4$ (large open space model). The shadowing was assumed to be log-normal distributed with a standard deviation of $\sigma = 5.8$ dB and a mean of 0 dB. For the time-selective fading, the maximum speed was chosen to be 1 m/s, while the power spectral density was chosen to have a Jakes-like shape typical for isotropic scattering models [6]. The multi-path propagation environment was characterized by a delay spread of $\Delta\sigma = 0.15\mu\text{s}$ with an exponential power delay profile. These chosen settings for path loss, shadowing and fading corresponded to the large open space model of ETSI C [7]. An example environment of such a setting would be a large airport or exposition hall, crowded with people.

4.2 Results

Figure 4.1 shows the capacity results achieved for $J = 6$ wireless terminals in the cell. The subcarrier fairness model generates almost always a slightly better average throughput per wireless terminal where the performance difference to the data-rate fair approach increases

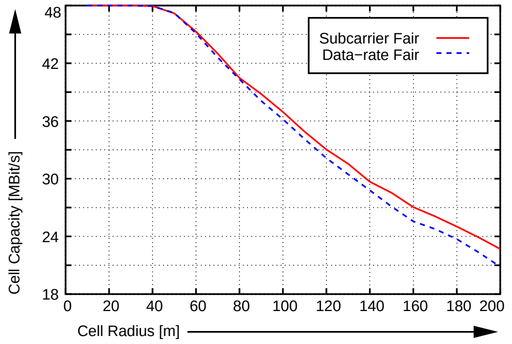

Figure 4.1: Capacity behavior for the two fairness models in the case of $J = 6$ wireless terminals in a cell with increasing radius

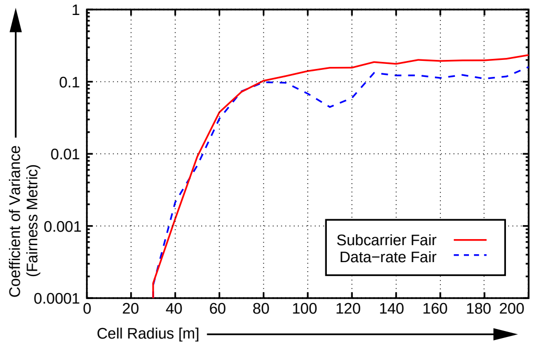

Figure 4.2: Fairness behavior for the two fairness models in the case of $J = 12$ wireless terminals in a cell with increasing radius

with the cell radius and decreases with the number of terminals in the cell. In fact, for $J = 24$ terminals in the cell the data-rate fair approach outperforms very slightly the subcarrier fair approach for some radius settings (not shown here).

In contrast to the behavior of the capacity, the two fairness models behave quite different in terms of the achieved throughput fairness. Figure 4.2 shows the fairness results for $J = 12$ terminals in the cell for both fairness approaches. While both models achieve almost the same fairness up to $R = 80$ m (with slight variations between both prior to that radius), for a bigger radius than 80 m the subcarrier fair model has an increasingly worse fairness value than the data-rate fair approach (which is not surprising, since it does not attempt to optimize for data-rate fairness at all). We call this largest radius for which the fairness measure of the subcarrier fair model equals the fairness measure of the data-rate fair model the *largest, fairness-equal radius*.

For an increasing number of terminals in the cell, the largest, fairness-equal radius increases – the subcarrier fair model actually manages to be data-rate fair as well for larger and larger cells. Contrarily, for a decreasing number of terminals in the cell this radius decreases.

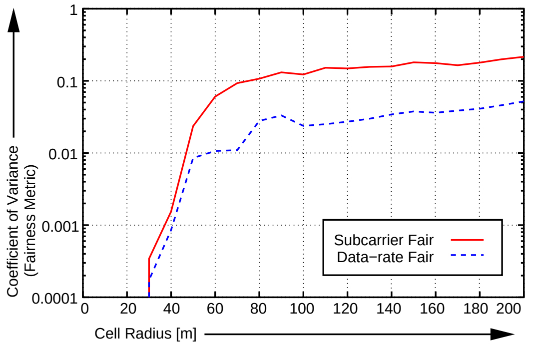

Figure 4.3: Fairness behavior for the two fairness models in the case of $J = 6$ wireless terminals in a cell with increasing radius

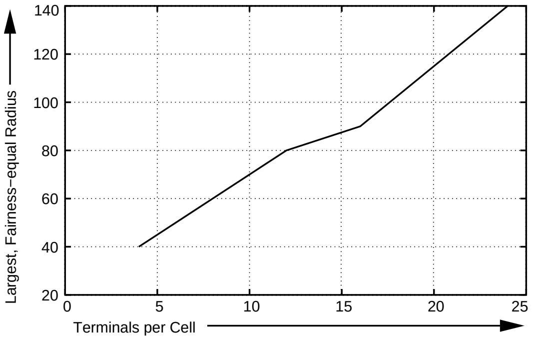

Figure 4.4: Largest, fairness-equal radius points of the two fairness approaches versus an increasing number of terminals in the cell

Figure 4.3 shows the fairness behavior for $J = 6$ terminals in the cell. In this case it can be observed that the subcarrier fair model is worse than the data-rate fair approach almost right away from a radius of 30 m. However, up to a radius of 50 m the fairness values of both approaches are separated by a constant offset and are steadily increasing. For a radius larger than 50 m this is not the case anymore, the subcarrier fair model's fairness values continue to increase while the data-rate fair approach achieves a slower increase. This behavior of the two curves is observed for 4, 6 and 8 terminals in the cell.

Considering this point where the data-rate fair model's curve starts to stabilize as the largest, fairness-equal radius for the cells with a lower number of terminals yields than the graph given in Figure 4.4. It shows the largest, fairness-equal radiuses depending on the number of terminals in the cell. For an increasing number of terminals in the cell these radii increase *linearly*.

This behavior is interpreted as follows: Up to a certain cell radius, depending on the number of terminals in the cell, it is not important which fairness model is used. Both models achieve almost the same fairness between all terminals in the cell considering average throughput provided to each terminal. For cells with a larger radius this is not true anymore. If data-rate fairness is to be granted for the terminals, using the subcarrier fair approach will no longer achieve this goal as good as the data-rate fair model does. Thus, using the data-rate fair approach is then the better choice.

Chapter 5

Conclusions and Future Work

In this paper we investigated the behavior of two different fairness models to be applied in dynamic OFDM-FDMA systems: A subcarrier fair model and a data-rate fair model. For both cases we studied the optimal solutions of linear programs which maximized the cell's capacity while providing either the subcarrier fairness or the data-rate fairness. Varying the number of terminals in the cell and the radius of the cell we simulated the performance of both approaches in terms of capacity and achieved throughput fairness, which we assume is the fairness each single user in a cell is more interested in.

In comparing both approaches we find that in terms of capacity the two models do not differ very much. The subcarrier fair approach achieves mostly a higher capacity in the cell than the data-rate fair model; the differences between the models are, however, mostly quite low. In contrast, considering the achieved throughput fairness of the two approaches shows that up to a certain radius both models achieve a comparable fairness between different terminals. For larger radii though, the subcarrier fair model becomes significantly more "unfair" than the data-rate fair model. This largest, fairness-equal radius depends linearly on the number of terminals in the cell.

As further work we consider two aspects. First it is of interest how these two approaches behave in terms of algorithmic complexity and measured run-time on real computers. This is an important aspect since we showed in this paper that up to a certain radius both approaches are quite comparable. If their complexity differs significantly then this would favor one approach over the other. As the subcarrier fair approach is likely to be more efficient, the practical consequence would then be that for cells up to certain size, the simpler subcarrier fair approach can be used while still delivering a basically fair allocation of data rate to the users.

The second aspect is to consider the impact of signaling. Obviously, signaling will decrease the performance of both algorithms; however, depending on the implementation, it might harm one of the used approaches more than the other. For example, the signaling overhead per terminal is always constant in the case of the subcarrier fairness but not so in the case of the data-rate fairness. Therefore, this aspect might also influence the choice of the model to be applied in a system.

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