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# Multiple Access Interference Mitigation in OFDMA Uplink using Dynamic Resource Allocation and Guard Bands

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## **Abstract**

In this report, proper frequency sub-band assignment and frequency guard band (GB) allocation is used to mitigate the effect of Multiple Access Interference (MAI), caused by synchronization offsets, in the uplink of OFDMA systems. This is done through formulation of an optimization problem, based on a solvable maximization of minimum user throughput. We can observe two main trade-offs. Firstly, if good frequency sub-bands is assigned to a badly-synchronized UT, the performance of that UT is improved but in the same time MAI on signals of other UTs is increased. Secondly, although MAI can be reduced by inserting GBs but inserting GBs implies the loss of resources and can lead to throughput loss due to this wastage. These two trade-offs are taken into account based on which a set of constraints for the optimization problem is defined. Adaptive coding modulation (ACM) is used to define the objective function of the optimization problem. The numerical evaluation of the proposed optimization algorithm shows an improvement of the minimum user throughputs by approximately 25% compared to the the estimation and correction based technique. Suboptimal resource allocation problems are considered where it is shown that the solving time can be reduced significantly by chunking adjacent subchannels.

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# Chapter 1.

## Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is one of the most popular techniques for wireless communication due to its robustness against fast fading and inter-symbol interference and its spectrum efficiency attained by orthogonality of subcarriers [4]. OFDM structure can be extended to multiple-access scenarios to include multiple user transmission. Orthogonal Frequency Division Multiple Access (OFDMA) is a multiple access technique based on OFDM which has been adopted in different standards such as IEEE 802.16 and Long Term Evolution (LTE). OFDMA is used in both downlink as well as uplink transmissions due to several of its favorable characteristics inherited from OFDM, such as efficient usage of spectrum, robustness against frequency selective fading and flexible resource allocation.

The primary challenge in usage of OFDMA in uplink is achieving synchronization between several User Terminals (UTs) and Base Stations (BS) in both time and frequency domain. The synchronization problem arises from the fact that signals arriving at BS in uplink are superpositions of user signals sent by several UTs simultaneously. Lack of synchronization disturbs the orthogonality and causes Multiple Access Interference (MAI) that can severely degrade performance of the system [5, 9]. In other words, MAI can be considered as the sum of Inter-Symbol Interference (ISI) caused by timing offsets and Inter-Carrier Interference (ICI) caused by frequency offsets.

The most common approach to deal with MAI is referred as the estimation and correction based technique (ECBT). As the name says, in these systems time and frequency offsets of UTs signals arriving at BS are estimated. These estimations are then used to subtract the offsets and to re-build orthogonality among subcarriers (see [6] and references therein). The estimation consists of two parts. First step is the coarse estimation which derives the offset through a rough estimation at the beginning of each uplink frame and it is normally based on the information embedded in the Cyclic Prefix (CP) [6]. The residual offsets on each OFDMA symbols of uplink frames are then estimated by a fine tracking process, which is build on the basis of pilot subcarriers. To alleviate the synchronization task, CP is recommended to be relatively long in order to deal with delay spread, 2-way-propagation delay and the timing errors caused by the asynchronicity of oscillators [8]. The main disadvantage of this approach lies in the large overhead and the significant reduction in throughput [15]. For example, corresponding to the Partially Used Sub-Channelization (PUSC) method in the IEEE 802.16e (WiMAX) standard, each uplink PUSC tile consists of 4 adjacent subcarriers in frequency and 3 symbols in time, and 4 out of the 12 subcarrier-symbol combinations (i.e. approximately 33% of system resources) are for pilot [8].

The question that arises here is whether one can reduce the overhead inflicted by long CP. One possible answer to this question is to resort to dynamic resource allocation for mitigating MAI and improving user throughput. In this approach, the system parameters can be dynamically adjusted to adapt better to the wireless channel condition and the performance goals. This approach stems from

the fact that MAI strongly depends on the assignment of frequency sub-bands to UTs, and is significantly reduced by suitable insertions of guards in the time and the frequency domains [9]. While guard in time or Guard Interval (GI) actually means the Cyclic Prefix (CP), a guard in frequency or Guard Band (GB) is an unmodulated frequency sub-band and assigned to no UTs. Therefore, exploiting the flexibility of resource allocation offered by OFDMA technique as well as the frequency and multi-user diversities is deemed to be promising to mitigate MAI and improve the UT performance.

In this report, we investigate in depth the mitigation of MAI and improvement of user throughputs in the OFDMA uplink via dynamic resource allocation. We try to examine the idea of using GBs in a scenario formulated according to assumptions close to reality. Roughly speaking, it is expected that the size of GB should be flexible such that it protects UTs individually from others. Therefore GBs are assigned dynamically according to the channel condition. Moreover it is shown here that the optimal resource allocation in OFDMA uplink can also mitigate MAI and improve the throughput. There are two main challenges in order to apply the dynamic resource allocation in OFDMA uplink and to deal with MAI and improve UT throughput. Firstly, assigning good frequency sub-bands to UT  $u$  might improve its throughput, but on the other hand, increases the MAI that UT  $u$  causes on other UTs' signals. This problem is considered as the trade-off of assignment. Secondly, there is another trade-off in using GB. Taking some interfering frequency sub-bands from a badly-synchronized UT  $u$  reduces the MAI it causes on other UTs' signals but leads to the loss of throughput of UT  $u$  and the resource wastage.

The problem of optimal resource allocation is studied by formulation of an optimization problem. The previously mentioned challenges in resource allocation should be taken into account in the formulation of the optimization problem. A novel optimization model based on a solvable maximization of minimum user throughput is introduced to solve the optimal resource allocation with aforementioned trade-offs. The optimal resource allocation assures the mutual damages caused by MAI are minimized, and the frequency and multi-user diversities are exploited. The original optimization problem is non-continuous (integer), non-linear, non-convex and NP-hard. We introduce several approaches to simplify the proposed OP in order to reduce the computational load and develop a practical solution for OFDMA-based systems.

The rest of this report is structured as follows. In Section 2, we describe the system model and main assumptions. In Section 3, the problem of dynamic resource allocation is addressed. A max-min user throughput optimization problem is formulated and various simplifications are discussed. The appendices present the proofs of main equations, and discuss the various numerical evaluation of preceding scenarios.

## Chapter 2.

# System Model

We consider one single cellular urban micro-cell consisting of one BS and  $M$  UTs. In the cell, OFDMA is used as data transmission scheme for downlink and uplink transmissions; the multiplexing method in use is Time Division Duplexing (TDD). The total available bandwidth  $B[H_z]$  is divided into  $N_{sca}$  subcarriers. Consequently, the subcarrier spacing is  $f_0 = B/N_{sca}$ , the OFDM symbol duration time takes a value of  $T_{sym} = 1/f_0$ , and the sampling interval equals  $T_{sam} = T_{sym}/N_{sca}$ . For each UT  $u$ , its relative time and frequency offsets relatively to BS are denoted by  $\tau_u$  and  $\Delta f_u$ , respectively. Let us assume that UTs always have data to send. Due to the frequency division multiple access (FDMA) scheme, UTs can take different unique subsets of total available subcarriers and send their data in parallel to BS through a multi-path fading channel. Let  $P_{k,u}^{TX}$  be the transmission power generated on subcarrier  $k$  by UT  $u$ . We assume that an equal and static transmission power  $P^{TX}$  for all subcarriers, it means  $P_{k,u}^{TX} = P^{TX}, \forall k, u$ . The number of OFDMA symbols in one uplink frame is denoted by  $N_{sym}$ .

The slow-fading channel in this paper is modeled reflecting path-loss, shadowing loss and multipath-loss. The selected models are listed as follows:

- Path-loss model: COST 231 Walfish-Ikegami with Non Line Of Sight (NLOS) (as recommended in [14]).
- Log-normal shadowing (as recommended in [14]).
- Multipath-loss: Clarke's model [13].
- The noise power is not explicitly modeled; the thermal noise density as adopted in the IEEE 802.16e standard [8] is used instead.

It is important to mention that during the downlink, in order to receive correctly the signals from BS, UTs needs to estimate the channel quality and the offsets. We assume these estimations are sent to BS. Consequently, BS presumably has perfect knowledge of the channel quality and time and frequency offset.

The signal arriving at BS is the superposition of attenuated, distorted, shifted in time and frequency constituents. In general, the received power on subcarrier  $k$  sent by UT  $u$  is  $P_{k,u}^{RX} = P^{TX} \times H_{k,u}$ , where  $H_{k,u}$  represents the  $u^{th}$  UT's average channel gain on subcarrier  $k$ . Similarly, the instantaneous Signal to Interference plus Noise Ratio (SINR)  $\gamma_{k,u}$  on the  $k^{th}$  subcarrier, which is uniquely assigned to UT  $u$  is given by

$$\gamma_{k,u} = \frac{P_{k,u}^{RX}}{\sigma^2 + \sum_{u' \neq u} \sum_{r \neq k} MAI_{k,u}^{r,u'}} \quad (2.1)$$

where  $\sigma^2$  is the noise power.  $MAI_{k,u}^{r,u'}$  is the average power of the MAI caused by subcarriers  $r$  of UT  $u'$  on subcarriers  $k$  of UT  $u$ . According to [9],  $MAI_{k,u}^{r,u'}$  is computed as follows:

$$MAI_{k,u}^{r,u'} = \frac{P_{r,u'}^{RX}}{N^2} \times \frac{A(k-r, \Delta f_{u'} - \Delta f_u, \tau_{u'} - \tau_u)}{\sin^2[\frac{\pi}{N}(k-r + \Delta f_{u'} - \Delta f_u)]} \quad (2.2)$$

where  $A(\cdot)$  is a function of the relative offset in time and frequency between UT  $u'$  and UT  $u$  as well as the distance between subcarriers  $k$  and  $r$ . The detailed calculation of  $A(\cdot)$  is presented in Appendix A.1 and it can be found in [9].

In order to avoid Inter-Symbol Interference (ISI), Cyclic Prefix (CP) is added in the time domain. To have a better insight, we assume CP consists of 2 parts, whose the lengths are  $\nu_1$  and  $\nu_2$ , respectively. The total OFDMA symbol duration time with CP is  $T = T_{sym} + \nu_1 + \nu_2$ . While  $\nu_2$  protects signals from the delay spread, the  $\nu_1$  part is to cope with two-way time offset caused by propagation delay and clock errors. Then ISI can be totally eliminated as long as the maximal time offset is smaller than  $\frac{1}{2}\nu_1$  and  $\nu_2$  is longer than maximal delay spread. It is recommended in IEEE 802.16m [8] to choose CP relatively long (e.g. 1/8 OFDMA symbol duration time). The MAI caused by skipping  $\nu_1$  as well as frequency offset is mitigated through the dynamic resource allocation.

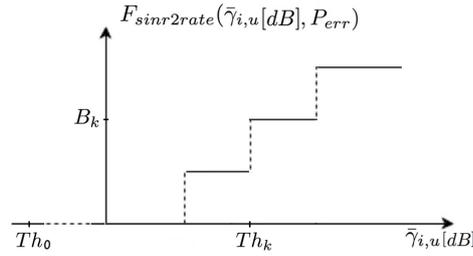


Figure 2.1.: Adaptive Coding and Modulation function  $F_{snr2rate}(\cdot)$

In the allocation process,  $G$  adjacent subcarriers are grouped into one subchannel in order to reduce the overhead for allocation addresses and the searching space of the optimization problem. The total subchannel number is denoted by  $Q = N_{sca}/G$ .  $K_u \subset Q$  is the unique subset of subchannel assigned to UT  $u$ . In this paper, the size of subchannel is selected equal to the coherent bandwidth of the wireless channel. Hence, we have  $H_{k,u} = H_{i,u}$  and, thus,  $P_{k,u}^{RX} = P_{i,u}^{RX} = P^{TX} \times H_{i,u}$  for all subcarrier  $k$  of subchannel  $i$ . Consequently, we define  $\overline{MAI}_{i,u}^{j,u'}$  the average MAI caused by subchannel  $j$  of UT  $u'$  on subchannel  $i$  of UT  $u$  as follows:

$$\overline{MAI}_{i,u}^{j,u'} = \frac{1}{G} \sum_{k \in i} \sum_{r \in j} MAI_{k,u}^{r,u'} \quad (2.3)$$

We derive the average SINR of subchannel  $i$  of UT  $u$ :

$$\tilde{\gamma}_{i,u} = \frac{P_{i,u}^{RX}}{\sigma^2 + \sum_{u' \neq u} \sum_{j \neq i} \overline{MAI}_{i,u}^{j,u'}} \quad (2.4)$$

We use Adaptive Coding and Modulation (ACM).

The adaptive coding and modulation (ACM), the Figure 2.1, is adopted and modeled by function  $F_{sindr2rate}(\tilde{\gamma}_{i,u}[dB])$ , which takes the instant SINR  $\tilde{\gamma}_{i,u}$  as input and returns the throughput of subchannel  $i$  of UT  $u$  subject to a predefined tolerable error rate  $P_{err}$ . The computed throughput equals the number of bits sent on subchannel  $i$  of UT  $u$  corresponding to the chosen ACM scheme divided by  $T$ . In total,  $L$  schemes are available. Let  $Th_{k,u,l}[dB]$  and  $B_{i,u,l}[bps]$  be the required SINR and the throughput of subchannel  $i$  of UT  $u$  when the ACM scheme  $l$  is chosen, respectively. For example, the 64-QAM with coding rate  $2/3$  scheme allows to send total 6 bits and ,thus, 4 data bits per OFDMA symbol; the throughput then equals  $4/T[bps]$ .

## Chapter 3.

# Dynamic Resource Allocation to Mitigate MAI

In order to apply the dynamic resource allocation approach to deal with MAI and improve the system performance, three main challenges need to be considered. First, as shown in (2.2), MAI is a function of not only relative time offset ( $\tau_u - \tau_{u'}$ ) and relative frequency offset ( $\Delta f_u - \Delta f_{u'}$ ), but also the distance ( $k - r$ ) between the interfering subcarrier  $r$  and the one under consideration  $k$ . Roughly speaking, MAI reduces as the distance increases (see Section B of the appendix). That means the mutual negative impact caused by MAI can reduce by choosing a "good arrangement".

Second, another main challenge is the trade-off of assignment, which is described as follows. On one hand, assigning a good subchannel  $i$  to UT  $u$  (i.e.  $H_{i,u}$  and, thus,  $P_{i,u}^{RX}$  is relatively high) obviously increases the SINR  $\gamma_{i,u}$  (as shown in (2.1)) and, thus, improves the throughput on subchannel  $i$  of UT  $u$ . But on the other hand, MAI caused by subchannel  $i$  of UT  $u$  on all subchannel  $j \neq i$  of all UT  $u' \neq u$  increases as  $P_{i,u}^{RX}$  increases (as shown in (2.2)). That means, improving throughput of UT  $u$  by giving it good subchannels might reduce throughput of all other UT and vice versa. Furthermore, we learn that, roughly speaking, optimizing the resource allocation based on only channel quality might lead to scenarios, in which the negative impact of MAI is increased. Hence, allocation process should consider both channel quality as well as mutual damage caused by MAI among subcarriers, thus the frequency and multi-user diversities can be better exploited in order to improve the UT throughput.

Moreover, it is shown in [9], [5] and [15] that MAI can be also efficiently mitigated by inserting frequency guard bands (GB). This is because of the fact that taking some subchannels from a badly-synchronized UT  $u'$  definitely reduces the MAI on all other subchannels of all UTs  $u \neq u'$ . However, although using GBs reduces MAI thereby improving throughput of UT  $u \neq u'$ , it leads to the wastage of frequency bandwidth and the loss of throughput. That fact gives rise to the second trade-off of using GB between throughput loss and MAI mitigation. An Optimization Problem (OP) is required to solve the aforementioned trade-offs and provide the optimal resource allocation to mitigate MAI and improve the UTs' throughput. In this work, the maximization of minimum user throughput (or shortly max-min user throughput) is chosen (as commonly used in literature).

### 3.1. Basic optimization problem

In this section, we show the general formulation for the max-min user throughput OP. The assignment of subchannel  $i$  to UT  $u$  is denoted by a binary optimization variable  $x_{i,u}$ , which takes 1 if UT  $u$  takes subchannel  $i$  and 0 if not. Then the general formulation of the max-min user throughput is described

as follows:

$$\begin{aligned}
 & \max \quad \varepsilon \\
 \text{s.t.} \quad & a) \quad \sum_{i=0}^{Q-1} [F_{\text{sinr2rate}}(\tilde{\gamma}_{i,u}[\text{dB}], P_{\text{err}}) x_{i,u}] \geq \varepsilon, \quad \forall u \\
 & b) \quad \sum_{u=0}^{M-1} x_{i,u} \leq 1, \quad \forall i
 \end{aligned} \tag{3.1}$$

where the instant SINR takes the form

$$\tilde{\gamma}_{i,u}[\text{dB}] = 10 \log_{10} \left( \frac{P_{i,u}^{\text{RX}}}{\sigma^2 + \sum_{u'=0, u' \neq u}^{M-1} \sum_{j \in K_{u'}} \overline{\text{MAI}}_{i,u}^{j,u'} x_{j,u'}} \right) \tag{3.2}$$

The first constraint in (3.1) assures that the all user throughputs are better than the lower threshold  $\varepsilon$ , which to be maximized. Constraint b) means a subchannel is assigned either to one UT, or left unmodulated and thus set as a GB. In general, OP in (3.1) is non-continuous (integer), non-linear, non-convex and NP-hard and there are three main mathematical difficulties in (3.1) listed as follows

- $F_{\text{sinr2rate}}(\cdot)$  in the first constraint of (3.1) is non-linear.
- Logarithm in (3.2) is non-linear.
- Optimization variables  $x_{r,u'}$  present in the denominator.

Therefore, OP1 cannot be solved directly by common optimizers (e.g. ILOG Cplex, Gurobi, ...).

### 3.2. Equivalent optimization problem

In order to transform OP in (3.1) to a solvable OP, we introduce new auxiliary continuous optimization variables  $\Omega_{i,u}$ , which stands for the throughput of subchannel  $i$  of UT  $u$ . Further, integer optimization variables  $z_{i,u,l}$  are used to represent the chosen ACM scheme, which takes 1 if scheme  $k$  is chosen for subchannel  $i$  of UT  $u$  and 0 if not. Then, OP in (3.1) can be transformed into the following form

$$\begin{aligned}
 & \max \quad \varepsilon \\
 \text{s.t.} \quad & a) \quad \sum_{i=0}^{N-1} x_{i,u} \Omega_{i,u} \geq \varepsilon, \quad \forall u \\
 & b) \quad \Omega_{i,u} \leq \sum_{k=0}^K z_{i,u,l} B_k, \quad \forall i, u \\
 & c) \quad \sum_{l=0}^L z_{i,u,l} \widetilde{\text{Th}}_{i,u,k} \geq \left( \sum_j \sum_{u'} \overline{\text{MAI}}_{i,u}^{j,u'} x_{j,u'} \right), \quad \forall i, u \\
 & d) \quad \sum_{l=0}^L z_{i,u,l} \leq 1, \quad \forall i, u \\
 & e) \quad \sum_u x_{i,u} \leq 1, \quad \forall i
 \end{aligned} \tag{3.3}$$

where  $\widetilde{Th}_k = (10^{(\tilde{\gamma}_{i,u} - Th_k)/10} - 1)$  and  $\tilde{\gamma}_{i,u}$  is the signal to noise (SNR) of subchannel  $i$  of UT  $u$ . See the Section A of the Appendix for the equivalent optimization problem. The max-min user throughput OP shown in (3.3), which is referred as OP1, is a non-convex Mixed Integer Quadratically Constrained Problem (MIQCP). Since the quadratical term  $x_{i,u}\Omega_{i,u}$  revolves binary variable  $x_{i,u}$ , OP1 is supposed to be transformed to a convex MIQCP and, thus, solved by an optimizer such as ILOG Cplex or Gurobi.

In general, since total number of subchannel is  $Q$  and each subchannel can be assigned to one of  $M$  UTs or set as GB, the total number of possibilities of allocation is computed as  $(M + 1)^Q$ . Take an example, one uplink frame in IEEE 802.16e consists of 16 subchannels shared among 4 UTs (assumably each UT takes one burst for the simplest case), then the searching space is  $(4 + 1)^{16}$  about more than 152 billions possibilities [8]. Therefore the optimization process might require a brutal computation.

### 3.3. Suboptimal optimization problems

First two approaches to simplify OP1 rise from its own mathematical principle. They can be listed as chunking and relaxation of integer optimization variables. To simplify OP1 with chunking,  $N_{ck}$  adjacent subchannels are grouped into a chunk. Consequently, we introduce the average MAI of chunk similar to (2.3) and then derive formulation of OP11 similar to (3.3) with the index  $i, j$  are now representing the chunk index. Although, chunking can provide suboptimal resource allocations, the searching space now is greatly reduced to  $(M + 1)^{Q/N_{ck}}$ . We denote the chunking solution by OP11. The second way is to relax some variables in OP1. To relax OP1, we introduce OP12, in which optimization variables  $\chi, \Omega, z$  are now defined as continuous variables, instead of integer as shown in (3.3). This might alleviate the difficulty of OP1 but leads to the sub-optimal resource allocation.

Furthermore, we develop 2 other heuristic OPs, which are shown in the following sub-sections.

#### 3.3.1. OP with fixed-width GBs

It can be easily seen that all the mathematical challenges mentioned in Section 4.1 can be avoided by replacing SINR by SNR and, thus, not considering MAI. By doing that, the OP in (3.1) can be simplified to a linear OP (referred as OP2) and, thus, easily coped with. However, since MAI is skipped, OP2 tends to insert no GBs. Consequently, the UT performance might be significantly reduce since the benefit of using GB is not exploited. Therefore, an extra constraint forcing GB in between UT data blocks is aided in OP2. In order to do that the optimization binary variable  $x_{i,u,m}$  has now 3 dimensions: subchannel  $i$ , UT  $u$  and transmission mode  $m$ .  $m$  takes 1 for user data and 0 for GB. The formulation of OP2 is shown in (3.4).

$$\begin{aligned}
 & \max \quad \varepsilon \\
 \text{s.t.} \quad & a) \quad \sum_{i=0}^{Q-1} [F_{sindr2bit}(\tilde{\gamma}_{i,u}[dB], P_{err})x_{i,u,1}] \geq \varepsilon, \quad \forall u \\
 & b) \quad \sum_{u=0}^{M-1} \sum_{m=0}^1 x_{i,u,m} = 1, \quad \forall i \\
 & c) \quad \text{if}(x_{i,u,1} = 1) \& (x_{i-1,u,1} = 0) \\
 & \quad \rightarrow x_{i-1,u,0} = 1, \quad \forall i, u
 \end{aligned} \tag{3.4}$$

where  $\tilde{\gamma}_{i,u}[dB]$  denotes the SNR and has the following form

$$\tilde{\gamma}_{i,u}[dB] = 10 \log_{10} \left( \frac{P_{i,u}^{RX}}{\sigma^2} \right) \quad (3.5)$$

As it can be seen from (3.4) and (3.5),  $\tilde{\gamma}_{i,u}[dB]$  is independent from optimization variables. OP2 is linear. Constraint b) assures a subchannel to be used only once, either for user data (i.e.  $x_{i,u,1} = 1$ ) or set as a GB (i.e.  $x_{i,u,0} = 1$ ). Since, MAI is not considered, the benefit of using GB is not included. Thus, Constraint c) forces one subchannel to be a GB between 2 adjacent signal blocks generated by 2 different UTs.

In general, ignoring MAI during the resource allocation process results in a linear OP2 but leads to 2 main drawbacks. First, only channel quality is considered in order to compute the resource allocation, thus, as mentioned above, the negative MAI might be increased. Second, since UT throughput is the function of SNR rather than SINR, the second drawback lies in the discrepancy between the UT throughput computed by OP2 and the one with consideration of MAI. See [5] for all the detail.

### 3.3.2. Minimization of minimum normalized MAI

Optimizing directly throughput with the consideration of MAI leads to very complex OPs. We develop a heuristic OP, whose the goal is to minimize the the mutual damage caused by MAI. However, although MAI on a subchannel might be reduced, such an approach can be inefficient as the corresponding received power on that subchannel is weak. In that case, the SINR in (2.4) is supposed to be really weak, and thus the throughput cannot be improved. Therefore, we come up with an OP, which minimizes the maximal sum of MAI normalized to the received power as shown following:

$$\begin{aligned} & \min \quad \phi \\ \text{s.t.} \quad & a) \quad \sum_{i=0}^{Q-1} \sum_{u' \neq u} \sum_{j \neq i} x_{i,u} x_{j,u'} \frac{MAI_{i,u}^{j,u'}}{P_{i,u}^{RX}} < \phi, \forall u \\ & b) \quad \sum_{u=0}^{M-1} x_{i,u} \leq 1, \forall i. \end{aligned} \quad (3.6)$$

The intuition is that by normalizing MAI to the received power, UTs are assigned to have subchannels, the received power on which are supposed to be much stronger than the MAI caused by its neighbors. However, since OP in (3.6) does not directly include the UTs' throughput, it is expected that no subchannels is actually assigned to UTs and all subchannels are set as GBs. To avoid this, a constraint assuring a minimum number of subchannels assigned to UTs is added. Then, we perform an iteration, in which the minimum number of subchannels assigned to UTs is increased in each loop as expressed by the constraint c). The formulation of the iteration of minimization of maximal sum of normalized MAI (referred as OP3) then has the form as follows

$$\begin{aligned}
 & \forall \Gamma \in \{1..Q/M\} \{ \\
 & \quad \min \quad \phi \\
 & \quad s.t. \quad a) \quad \sum_{i=0}^{Q-1} \sum_{u' \neq u} \sum_{j \neq i} x_{i,u} x_{j,u'} \frac{MAI_{i,u}^{j,u'}}{P_{i,u}^{RX}} < \phi, \forall u \\
 & \quad \quad b) \quad \sum_{u=0}^{M-1} x_{i,u} \leq 1, \forall i \\
 & \quad \quad c) \quad \sum_{i=0}^{Q-1} x_{i,u} \geq \Gamma, \forall u \\
 & \quad \}
 \end{aligned} \tag{3.7}$$

By normalizing MAI to the corresponding received power, it is expected that the sum of MAI caused by UTs on subchannel  $i$  is decreased, and at the same time UTs tend to take subchannels, whose channel conditions are relatively good. This supposedly leads to an improvement in throughput. As it can be seen from (3.7), OP3 is a Mixed-Integer Quadratically-Constrained Quadratic Program (MI-QCP), and can be solved by MI-QCP optimization solvers like ILOG's CPLEX. There are  $Q(Q-1)M$  quadratic terms in the first constraint of OP5 for each UT.

## Chapter 4.

### Conclusion

It was shown that to use the concept of GBs, the proper subcarrier allocation scheme with frequency guard assignment should be conducted. Users with synchronization problems should be assigned to weak channels to reduce their MAI however such an allocation scheme can violate fairness among users. Moreover frequency guard bands, in spite of MAI mitigation, involve loss in user throughputs. To deal with preceding facts, we formulated a max-min optimization problem and provided simplifications of the OP to reduce solving time. It was shown by numerical results that the system throughput, defined as maximum of minimum user throughput, can be significantly improved by proper resource allocation.

## Appendix A.

## Appendix

### A.1. Calculation of MAI

Multiple access interference can be calculated as follows:

$$MAI_{k,u}^{r,u} = \frac{P_{r,u}^{RX}}{N^2} \frac{A(k-r, \Delta f_u, \tau_u)}{\sin^2(\frac{\pi}{N}(k + \Delta f_u - r))} \quad (\text{A.1})$$

where  $A(k-r, \Delta f_u, \tau_u)$  is computed as follows

- Case 1:  $(-v_1/2 - v_2) > \tau_u \geq (-N + v_1/2)$ :

$$A = \sum_{l=0}^{L_u-1} \Omega_l^u \left[ \sin^2 \frac{\pi}{N} (l - \tau_u - \frac{v_1}{2} - v_2) (\Delta f_u + k - r) \right. \\ \left. + \sin^2 \frac{\pi}{N} (l - \tau_u - \frac{v_1}{2} - v_2 - N) (\Delta f_u + k - r) \right]$$

- Case 2:  $(L_u - 1 - v_1/2 - v_2) > \tau_u \geq (-v_1/2 - v_2)$ :

$$A = \sin^2(\pi \Delta f_u) \sum_{l=0}^{v-|\tau_u|} \Omega_l^u \quad (\text{A.2}) \\ + \sum_{l=v-|\tau_u|+1}^{L_u-1} \Omega_l^u \left[ \sin^2 \frac{\pi}{N} (l - \tau_u - \frac{v_1}{2} - v_2) (\Delta f_u + k - r) \right. \\ \left. + \sin^2 \frac{\pi}{N} (l - \tau_u - \frac{v_1}{2} - v_2 - N) (\Delta f_u + k - r) \right]$$

- else if  $-v_1/2 \geq \tau_u \geq (L_u - 1 - v_1/2 - v_2)$  then:

$$A = \sin^2(\pi \Delta f_u) \sum_{l=0}^{L_u-1} \Omega_l^u$$

- else if  $L_u - 1 + v_1/2 \geq \tau_u > v_1/2$  then:

$$A = \sin^2(\pi \Delta f_u) \sum_{l=|\tau_u|+1}^{L_u-1} \Omega_l^u \quad (\text{A.3}) \\ + \sum_{l=0}^{|\tau_u|} \Omega_l^u \left[ \sin^2 \frac{\pi}{N} (l - \tau_u + \frac{v_1}{2}) (\Delta f_u + k - r) \right. \\ \left. + \sin^2 \frac{\pi}{N} (l - \tau_u + \frac{v_1}{2} + N) (\Delta f_u + k - r) \right]$$

- else if  $(N + v_1/2) \geq \tau_u > (-L_u - 1 + v_1/2)$  then:

$$A = \sum_{l=0}^{L_u-1} \Omega_l^u \left[ \sin^2 \frac{\pi}{N} \left( l - \tau_u + \frac{v_1}{2} \right) (\Delta f_u + k - r) \right. \\ \left. + \sin^2 \frac{\pi}{N} \left( l - \tau_u + \frac{v_1}{2} + N \right) (\Delta f_u + k - r) \right]$$

Table A.1.: System parameters.

Parameters	Values
Number of UTs	4
Number of subcarriers	128
Subcarrier spacing	10940 Hz
Number of subcarriers per subchannel	8
Number of subchannels	16
Number of subchannels per chunk	2
Power per subcarrier	1 mW
MS maximal time offset	25 $\mu s$
MS maximal frequency offset	2000 Hz
Cell radius	250 m
Path loss model	COST231 Walfish-Ikegami
Log normal shadowing std. dev.	10 dB
Multipath fading model	Jakes's/Clarke's model
Penetration and other losses	10 dB
Delay spread (rms)	0.251 $\mu s$
Receive Antenna Gain	14 dB
$v_2$	equal the maximal delay spread
$v_1$	0 $\mu s$

## A.2. Numerical Performance Evaluation for Dynamic Resource Allocation

To numerically evaluate the performance of the proposed approach, we have developed a simulation of the system described in Section 2 on the network simulation framework OMNeT++ using the MiXiM library. The parameterization selection is based on the IEEE 802.16e standard [8]. The values of parameters are chosen according to [8] and [12], and shown in Table A.1.

Let us adopt the root mean square of the delay spread  $\sigma_\tau = 0.251 \mu s$  as recommended in [14] and the frequency correlation function over the coherence bandwidth  $B_c$  is above 0.9. Then corresponding to [11], we have  $B_c \approx \frac{1}{50\sigma_\tau}$ , which equals 7.968 [kHz]. Therefore a subchannel assumably consists of 8 adjacent subcarriers.

Corresponding to Table 8-3 in Page 803 of Standard IEEE802.16m, the maximal time offset of UTs is chosen equal 25% of the symbol duration (i.e.  $T_{sym}$ , and the transmit frequency accuracy is  $\pm 1 \times 10^{-6}$  [Hz]. Assume the centre frequency is  $f_c = 2.5$  [GHz], we derive the maximal frequency offset equal  $2.5 \times 10^3$  [Hz], which approximately equals 0.2 frequency spacing.

We simulate in total 20 runs, and total 100 uplink frames per run are conducted. And for each uplink frame, the UTs are dropped uniformly in the cell. Apart from the proposed OPs, we also simulate the heuristic OPs introduced in [5] (referred as OP2) and [15] (referred as OP3) as reference. While OP2 is built based on the SNR instead of SINR (and MAI is thus not considered), OP3 aims to minimize the maximum of normalized MAI. During each uplink frame, at the same system status, instances of the optimization OP1, OP11, OP12 and OP3 are formed and sent to the Gurobi OP solver. Since OP2 contains logical expressions, it could not be solved by Gurobi OP solver, ILOG Cplex as

chosen in the original work is used to solve OP2 instances. Resource allocation achieved at the output of the optimization solver are used to calculate UT throughputs.

To compare the performance achieved by the proposed OPs with the one of ECBT, a simple system, based on the IEEE 802.16m standard [8], with long CP together with pilot subcarriers to deal with MAI is simulated. In this system, first, the CP is assumed to be sufficient to deal with delay spread, propagation delay and clock errors, and hence ISI is thus fully mitigated. The pilot subcarriers are inserted following the Partially Used Sub-Channelization (PUSC) method in the IEEE 802.16m [8]. It means 33% of overall subcarriers are pilots. Finally, the block-wise strategy is adopted to statically assign subchannels to UTs. We evaluate two cases, referred as ECBT1 and ECBT2. First, the estimation and correction is assumed to be perfect in ECBT1, thus no time and frequency offsets exist. Second, there is however always a chance that estimation is not perfect [7], which leads to a residual frequency offset. Thus, in ECBT2, we assume the residual frequency offset is  $10\text{Hz}$ . Note that the Doppler shift for the velocity of  $30\text{km/h}$  is about  $70\text{Hz}$ . It is important to mention that results for the simple ECBT system and the proposed OPs are derived from exact the same system status (e.g. wireless channel, offset profile).

Figure B.1 shows the minimum user throughput achieved by different OPs as well as the simple system based on ECBT. The two most left columns illustrates the comparison of OP1 and ECBT1 in the idealized scenario. As it can be seen, OP1 provides a significant gain (about 26%) compare to ECBT1. On the right, under the assumption of the residual offset according to the literature, applying dynamic resource allocation offers the potential to improve the minimum user throughput by 25% compared to ECBT2. Furthermore, among resource allocation optimization, the novel OP1 provides a great improvement compared to OP2 and OP3 equivalent to about 300% and 129%, respectively. Therefore by considering channel quality and MAI, the system performance can be greatly improved subject to the desired goal. The chunking approach OP11 and the concurrent OP13 also provide improvement (190% and 115% compared to OP2 and OP3, respectively) but the relaxation of OP variables in OP12 leads to poorer performances.

With regard to the solving time, Figure B.2 shows the quartiles of solving time for different optimizations. (Because the distributions of solving time have heavy tails, mean values with confidence intervals are replaced by quartiles.) As it shows, although OP11 provides suboptimal resource allocation, it could be solved much faster than OP1. The median value of OP11 is less than 0.1s equal 10% of the one of OP1.

To have an insight, we quantize the fragmentation of resource allocation by investigate number of GBs and number of Heterogeneous Junctions, which is defined as the border between 2 user data blocks assigned to 2 different UTs. First, Figure B.3 shows the average number of GBs inserted when different OPs are used. Furthermore, the histogram of number of GBs of OP1 is shown in Figure B.4. As it can be seen from Figure B.3, B.4 and B.5, OP1 improves the minimum user throughput as expected by exploiting better the frequency and multi-user diversities.

### A.3. Equivalent optimization problem with OP1

This section shows a mathematical transformation to avoid the 3 aforementioned challenges. In order to do that, first,  $\Omega_{i,u} = F_{\text{sinr2rate}}(\gamma_{i,u}[\text{dB}], P_{\text{err}})$  becomes now an auxiliary continuous optimization variables. Second, we re-consider the formulation of the ACM function  $F_{\text{sinr2rate}}(\gamma_{i,u}[\text{dB}], P_{\text{err}})$ , whose meaning can be essentially represented in the Figure 2.1.

In other words, for each subchannel  $i$  and UT  $u$ , we have

$$\begin{aligned}
 & \text{for } (\forall k \in K) \\
 & \quad \text{if } (Th_k < \gamma_{i,u}) \quad \text{then } \Omega_{i,u} = B_k \\
 & \quad \text{end}
 \end{aligned} \tag{A.4}$$

Further, we have

$$\begin{aligned}
 & Th_k \leq \gamma_{i,u} [dB] \\
 \Leftrightarrow & \quad Th_k \leq 10 \log_{10} \left( \frac{P_{i,u}^{RX}}{\sigma^2 + \sum_{j,u'} MAI_{i,u}^{j,u'}} \right) \\
 \Leftrightarrow & \quad \tilde{Th}_k \geq \sum_{j,u'} \frac{MAI_{i,u}^{j,u'}}{\sigma^2}
 \end{aligned} \tag{A.5}$$

where

$$\tilde{Th}_k = 10^{(\tilde{\gamma}_{i,u} - Th_k)/10} - 1 \tag{A.6}$$

and  $\tilde{\gamma}_{i,u}$  is the Signal to Noise Ratio (SNR) of subchannel  $i$  of UT  $u$  and takes the form  $\tilde{\gamma}_{i,u} = P_{i,u}^{RX} / \sigma^2$ .

We then introduce auxiliary integer optimization variable  $z_{i,u,l}$  to represent the ACM scheme, which takes 1 if scheme  $k$  is chosen for subchannel  $i$  of UT  $u$ . To make the mathematical transformation valid, it is necessary to add an ACM scheme (the first one) representing the case when  $\gamma_{i,u}$  is too small to use any modulation; it means  $B_0 = 0$  when  $\tilde{Th}_0 \geq \gamma_{i,u} < \tilde{Th}_1$ .  $Th_0$  is a sufficiently large constant; it has to be larger than all possible values of  $\sum_j \sum_{u'} MAI_{i,u}^{j,u'} x_{j,u'}$ . Then (A.4) can be written as

$$\Omega_{i,u} = \sum_{k=0}^K z_{i,u,l} B_k \tag{A.7}$$

together with

$$\sum_{k=0}^K z_{i,u,l} \tilde{Th}_k \geq \sum_{j,u'} \frac{MAI_{i,u}^{j,u'}}{\sigma^2} \tag{A.8}$$

Finally, we have the formulation as shown in (3.3).

## Appendix B.

### List of Figures

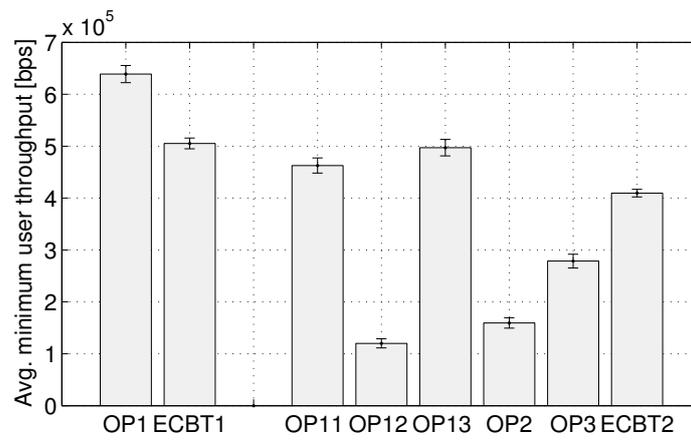


Figure B.1.: Avg. of minimum user throughput

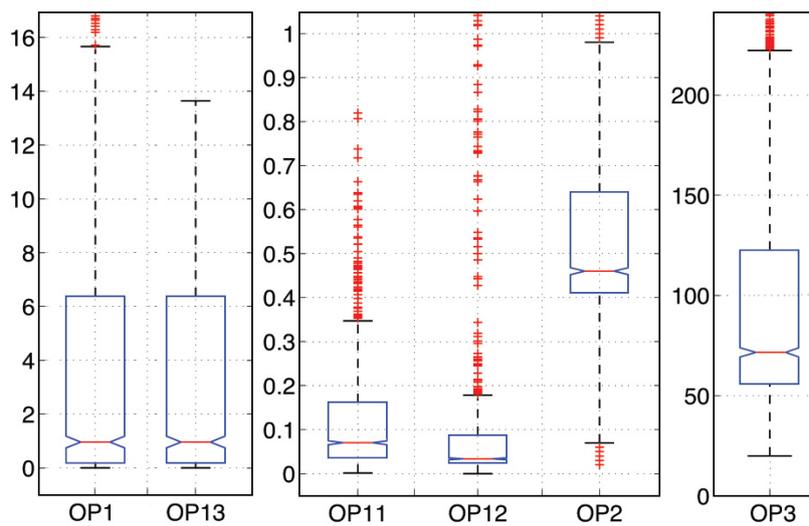


Figure B.2.: Quartiles of solving time in second

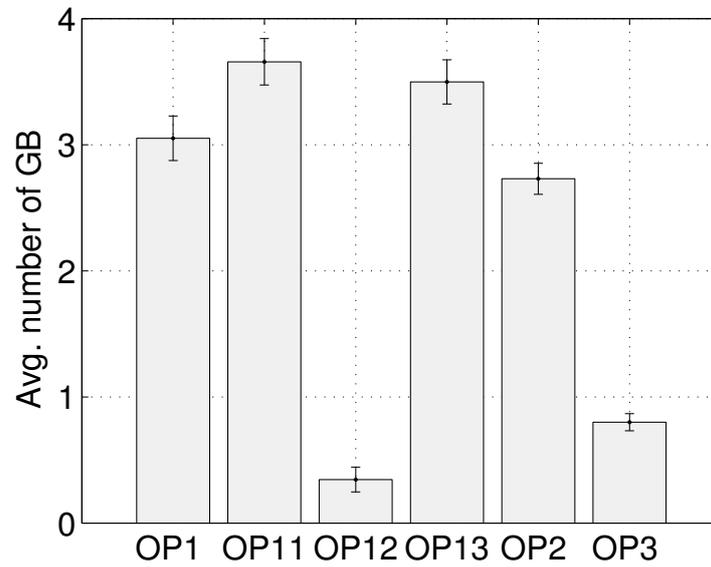


Figure B.3.: Avg. number of GBs

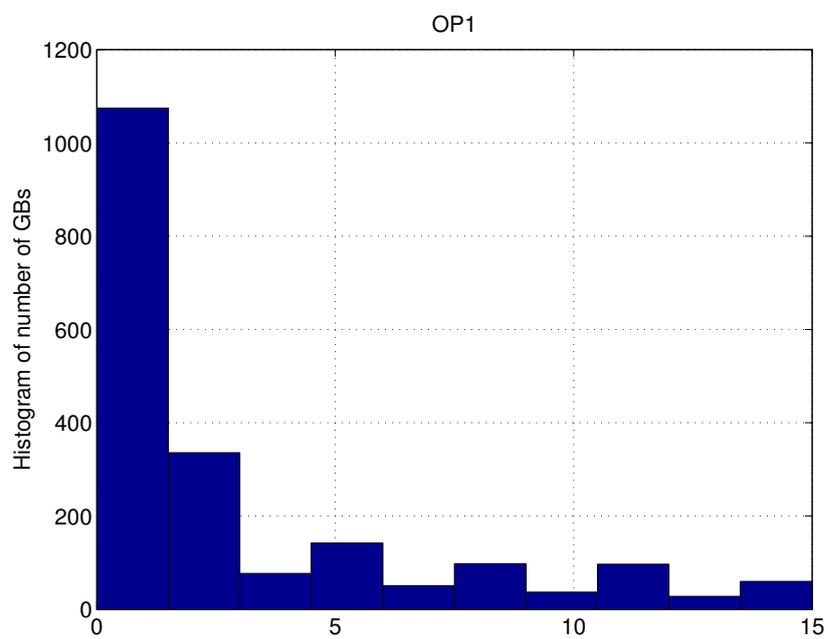


Figure B.4.: Histogram of number of GBs of OP1

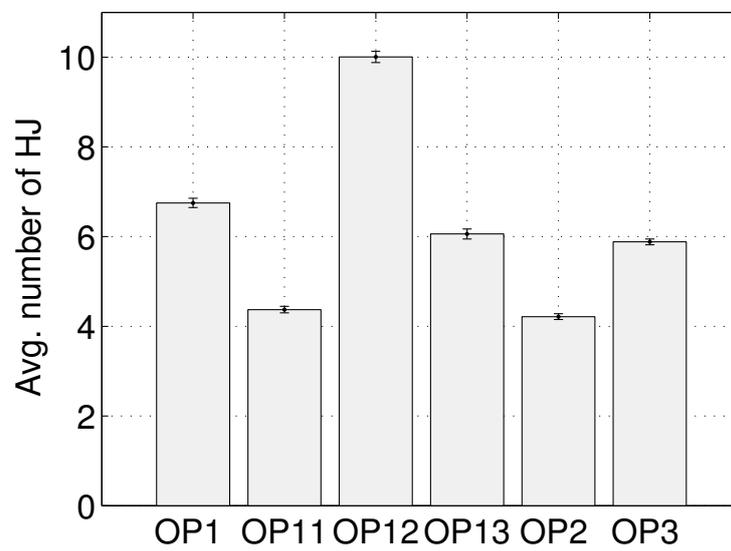


Figure B.5.: Avg. number of HJs

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